Journal of Engineering Physics and Thermophysics, Vol. 81, No. 6, 2008

NONLINEAR VIBRATIONS OF HYDRAULIC SHOCK ABSORBERS

N. A. Dokukova,[a] **M. D. Martynenko,**[b] **and E. N. Kaftaikina**[c]

UDC 517.9

A method of equivalent linearization for a nonlinear dynamic system of vibrations with a hydraulically elastic shock absorber is presented. An integral estimation of the exact and approximate solutions is obtained. It is shown that with an appropriate choice of the parameter λ the linearized dynamic system differs insignificantly from the nonlinear one. The influence of nonlinear factors on the coefficients of the dynamic rigidity and transmission has been established. These results agree well with the experimental curves published in the foreign scientific literature.

Introduction. Designing of hydraulically elastic shock absorbers with prescribed functional properties that considerably improve the vibrational protective characteristics of mobile machines is associated with the solution of nonlinear dynamic systems unsolvable in their final form. It is worthwhile that qualitative investigation of such a mechanical model be made by approximate methods that allow one to establish the laws governing its parameters on a theoretical level. For this purpose we will use the general method of a numerical-analytical investigation of vibrational processes for the solution of nonlinear differential equations developed by the present authors in [1–3].

Statement of the Problem. The general system of nonlinear differential equations of the dynamics of a hydraulic shock absorber has the form

$$\ddot{x}_1 = 2n_1(\dot{x}_2 - \dot{x}_1) + \omega_1^2(x_2 - x_1) - f_1(1-\delta)\cos\omega t + \eta\,\mathrm{sign}(\dot{x}_2 - \dot{x}_1)(\dot{x}_2 - \dot{x}_1)^2\,,$$
$$\ddot{x}_2 = 2n_2(\dot{x}_1 - \dot{x}_2) + \omega_3^2(x_1 - x_2) - \omega_2^2 x_2 + f_2\cos\nu t - kf_1\delta\cos\omega t - k\eta\,\mathrm{sign}(\dot{x}_1 - \dot{x}_2)(\dot{x}_2 - \dot{x}_1)^2\,. \qquad (1)$$

Analytical Solution. Generally the system of equations (1) cannot be integrated. To obviate this feature of the system, we will use the method of equivalent linearization to derive simple analytical relations; following this method, the function $\eta\,\mathrm{sign}\,(\dot{x}_1 - \dot{x}_2)(\dot{x}_2 - \dot{x}_1)^2$ is linearized by means of replacement

$$\eta\,\mathrm{sign}(\dot{x}_1 - \dot{x}_2)(\dot{x}_2 - \dot{x}_1)^2 \to K_2\left(\dot{\tilde{x}}_2 - \dot{\tilde{x}}_2^0 - \dot{\tilde{x}}_1 + \dot{\tilde{x}}_1^0\right) + K_1(\tilde{x}_2 - \tilde{x}_1)\,, \qquad (2)$$

$$K_1 = \frac{\eta\,\mathrm{sign}(\dot{x}_2^0 - \dot{x}_1^0)(\dot{x}_2^0 - \dot{x}_1^0)^2}{x_2^0 - x_1^0} \equiv \frac{r(0)}{x_2^0 - x_1^0}\,, \qquad (3)$$

$$K_2 = K_1\,, \qquad (4)$$

Here, x_1^0, $\dot{x}_1^0$, x_2^0, and $\dot{x}_2^0$ are $x_1(0)$, $\dot{x}_1(0)$, $x_2(0)$, and $\dot{x}_2(0)$, $r(t) = \eta\,\mathrm{sign}\,(\dot{x}_2 - \dot{x}_1)(\dot{x}_2 - \dot{x}_1)^2$. The condition $x_2^0 - x_1^0 \neq 0$ is achieved when as the initial values of x_1^0 and x_2^0 we select the conditions of stable motion of the stationary solution of the system of differential equations (1). At the initial moment at $t = 0$ the stationary values x_1 and x_2 will be as follows:

[a]International Institute of Labor and Social Relations, 21/1 Kazinets Str., Minsk, 220099, Belarus; email: dokukova@mail.ru; [b]Belarusian State University, 4 Nezavisimost' Ave., Minsk, 220050, Belarus; [c]Joint Institute of Machine Building, National Academy of Sciences of Belarus, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 81, No. 6, pp. 1154–1157, November–December, 2008. Original article submitted September 26, 2007; revision submitted November 13, 2007.

1062-0125/08/8106-1197©2008 Springer Science+Business Media, Inc.

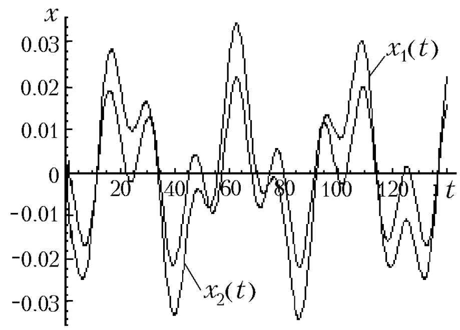

Fig. 1. Vibrations $x_1(t)$ and $x_2(t)$ of a hydraulic support. t, sec; x, m.

$$x_1(0) = \frac{f_2}{\omega_2^2} + f_1\left(\frac{1-\delta}{\omega_1^2}\left(k\frac{\omega_1^2}{\omega_2^2} - 1\right) - \frac{k\delta}{\omega_2^2}\right), \quad x_2(0) = \frac{f_2}{\omega_2^2} + \frac{f_1}{\omega_2^2}\left(\frac{\omega_3^2}{\omega_1^2}(1-\delta) - k\delta\right), \quad x_2^0 - x_1^0 = \frac{f_1(1-\delta)}{\omega_1^2}. \tag{5}$$

For efficient vibrational protection of a controlled object all the parameters must satisfy the stability condition. Such an analysis is made by compiling the characteristic equation of a linearized system of equations in variations and with the use of the Hurwitz criterion [4, 5]. The discriminant of the considered mechanical system is negative, the pair of complex roots has negative real parts, and the other two roots are real negative. The stable node focus is a singular point; all the trajectories of the sought solutions are wound as a spiral. The vibrations $x_1(t)$ and $x_2(t)$ of the hydraulic support are stable (Fig. 1).

The initial conditions for the exact and linearized solutions are selected according to the procedure developed in [1, 2]. We assume that $\dot{x}_2^0 - \dot{x}_1^0 \neq 0$. The solution of the system of nonlinear differential equations (1) leads to the solution of the linear system of ordinary differential equations:

$$\ddot{\tilde{x}}_1 = 2n_1(\dot{\tilde{x}}_2 - \dot{\tilde{x}}_1) + \omega_1^2(\tilde{x}_2 - \tilde{x}_1) - f_1(1-\delta)\cos\omega t + K_2\left(\dot{\tilde{x}}_2 - \dot{\tilde{x}}_2^0 - \dot{\tilde{x}}_1 + \dot{\tilde{x}}_1^0\right) + K_1(\tilde{x}_2 - \tilde{x}_1),$$
$$\ddot{\tilde{x}}_2 = 2n_2(\dot{\tilde{x}}_1 - \dot{\tilde{x}}_2) + \omega_3^2(\tilde{x}_1 - \tilde{x}_2) - \omega_2^2\tilde{x}_2 + f_2\cos\nu t - kf_1\delta\cos\omega t - k\left(K_2\left(\dot{\tilde{x}}_2 - \dot{\tilde{x}}_2^0 - \dot{\tilde{x}}_1 + \dot{\tilde{x}}_1^0\right) + K_1(\tilde{x}_2 - \tilde{x}_1)\right). \tag{6}$$

Integration of (6) presents no problems.

Error Estimation. The problem of the accuracy of the approximate solution remains unclear. To answer it, one must estimate the error between the exact and approximate solutions:

$$z_1(t) = x_1(t) - \tilde{x}_1(t), \quad z_2(t) = x_2(t) - \tilde{x}_2(t). \tag{7}$$

Let us substract Eqs. (6) respectively from the system of equations (1) and estimate the resulting expressions by the norm on the basis of the Lipshits conditions for continuous functions over $[0, T]$ with a choice of the arbitrary parameter λ and constants A_* and B_* from the following conditions:

$$A_* = \max_{t, x_1^0, x_2^0, \dot{x}_1^0, \dot{x}_2^0}\left\{2n_1 + |K_2|, \omega_1^2 + |K_2|\right\}, \quad B_* = \max_{t, x_1^0, x_2^0, \dot{x}_1^0, \dot{x}_2^0}\left\{M + |K_2|, |K_2|\right\} = M + |K_2|,$$
$$C(\lambda, T) < \frac{2}{(k+1)A_* + kB_*}. \tag{8}$$

The errors between exact solutions (1) and approximate ones (6) do not exceed the following values:

$$\|z_1\|_\lambda \le \frac{B_* C(\lambda, T)}{2 - A_*(k+1)C(\lambda, T)}\left(\|x_1 - x_1^0\|_\lambda + \|x_2 - x_2^0\|_\lambda\right),$$

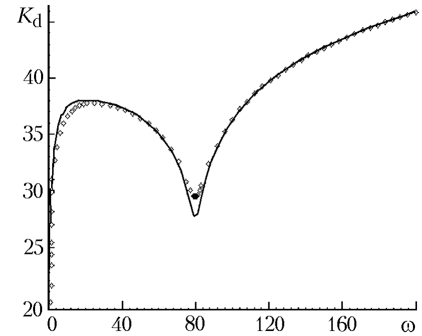

Fig. 2. Coefficient of dynamic rigidity K_d of a hydraulic support in the linear model (solid curve) and nonlinear one (dots). K_d, dB; ω, rad/sec.

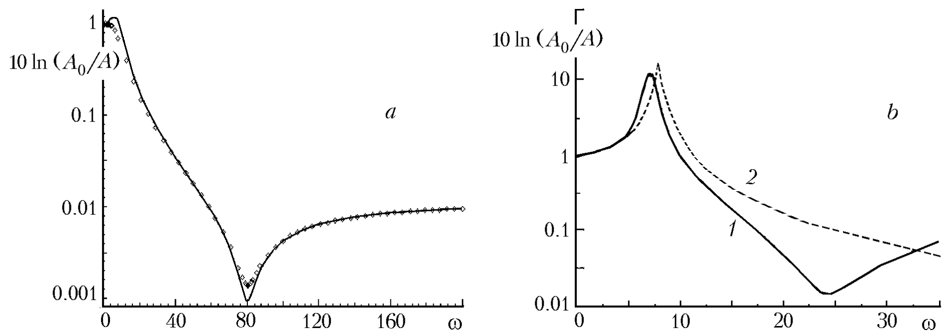

Fig. 3. Logarithmic dependences of the coefficient of hydraulic support transmission: a) in a linear model (solid curve) and nonlinear one (dots); b) experimental curves: of the transmission coefficient of hydraulic support (1) and of an elastic shock-absorber (2). ω, rad/sec.

$$\|z_2\|_\lambda \le \frac{kB_* C(\lambda, T)}{2 - A_*(k+1)\,C(\lambda, T)}\left(\|x_1 - x_1^0\|_\lambda + \|x_2 - x_2^0\|_\lambda\right). \tag{9}$$

The method of equivalent linearization allows one to investigate vibrational processes in nonlinear mechanical systems of general form, since the above-given explicit solutions of linearized equations and estimates prove their closeness to the sought exact solutions with a corresponding selection of the parameter λ. Therefore the proposed linearization of (6) allows one to represent the nonlinear differential equations of the vibrations of a hydraulically elastic shock absorber in a form convenient for an analysis in order to elucidate the influence of nonlinear factors on the mechanical system as a whole. With allowance for the well-known relations $n_2 = kn_1$, $\omega_3^2 = k\omega_1^2$, $k = m_1/m_2$ for the parameters, the system of differential equations can be rewritten as follows:

$$\ddot{x}_1 = \left(2n_1 - |K_2|\right)(\dot{x}_2 - \dot{x}_1) + \left(\omega_1^2 - |K_1|\right)(x_2 - x_1) - f_1(1-\delta)\cos\omega t + |K_2|\left(\dot{x}_2^0 - \dot{x}_1^0\right), \tag{10}$$

$$\ddot{x}_2 = k\left(2n_1 - |K_2|\right)(\dot{x}_1 - \dot{x}_2) + k\left(\omega_1^2 - |K_1|\right)(x_1 - x_2) - \omega_2^2 x_2 + f_2\cos\nu t - kf_1\delta\cos\omega t - k\,|K_2|\left(\dot{x}_2^0 - \dot{x}_1^0\right).$$

As is seen, the damping properties and the squared frequency of the hydraulic support change by constant values K_2 and K_1, and they increase or decrease depending on the sign of the function sign $(\dot{x}_1 - \dot{x}_2)$. This leads to a change in the dynamic characteristics of the hydraulic support: dynamic rigidity and coefficient of transmission. Therefore it is worthwhile to seek their optimal values for each specific mechanism.

The decrease in the frequency of the natural vibrations of the upper chamber by the value of K_1 insubstantially influences the dynamic parameters of the hydraulic support; the decrease in the coefficient of damping $2n_1$ by

K_2 substantially decreases the dynamic rigidity K_d of the hydraulic support in the one of the first resonance and raises it in the zone of the second resonance (Fig. 2).

A decrease in the magnitudes of the dissipative hydraulic resistances leads to a decrease in the transmission of the perturbing forces from the power-generating set at antiresonance and to an increase in the dynamicity at the second resonance (Fig. 3a). The form of the curves in Fig. 3a is similar to the experimental curves available in the foreign and Russian periodic scientific literature (Fig. 3b).

Conclusions. The effect of the "inertial transformer" of a hydraulic insulator [6] as a result of which frequency regions of increased damping of vibrations are created due to internal resonances of the two-chamber system has been established experimentally by Russian scientists. Technically this means that into the partition between the chambers with liquid a solid-body inertia element is introduced that acts as a "negative" rigidity and decreases the relative damping and the general coefficient of transmission at necessary frequencies. In the present work this fact is confirmed by theoretical solutions of systems of nonlinear differential equations by the method of equivalent linearization.

NOTATION

A_0, A, initial and maximum values of the vibration amplitude $x_1(t)$, m; A_*, B_*, constant numbers; f_1, f_2, vibrational accelerations, m/sec^2; k, coefficient of the ratio of masses m_1/m_2; K_1, K_2, coefficients of linearization; K_d, coefficient of dynamic rigidity, dB; M, Lipshits constant; m_1, m_2, masses of bodies, kg; n_1, n_2, physical constants, rad/sec; T, time, sec; T, limiting value of the time parameter, sec; x_1, x_2, exact solutions, m; $\tilde{x}_1$, $\tilde{x}_2$, approximate solutions relative to points x_1, x_2, m; $z_1(t)$, $z_2(t)$, errors between exact and approximate solutions, m; δ, λ, dimensionless parameters; η, physical constant, 1/m; ν, frequency of external force fluctuations, rad/sec; ω, variable frequency, rad/sec; ω_1, ω_2, ω_3, frequencies of conservative vibrational systems, rad/sec. Subscript: d, dynamic.

REFERENCES

1. N. A. Dokukova and M. D. Martynenko, Linearization method and its application to the analysis of nonlinear vibrational systems, *Vestsi Nats. Akad. Navuk Belarusi, Ser. Fiz.-Tékh. Navuk*, No. 3, 108–111 (1999).
2. M. D. Martynenko, N. A. Dokukova, and L. I. Boiko, Vibrational processes in mechanical systems, *Inzh.-Fiz. Zh.*, **72**, No. 3, 491–494 (1999).
3. N. A. Dokukova and M. D. Martynenko, Linearization in nonstationary problems of the bending of girders at the nonlinear law of elasticity, *Vestsi Akad. Navuk BSSR, Ser. Fiz.-Tékh. Navuk*, No. 1, 38–43 (1988).
4. N. I. Levitskii, *Oscillations in Mechanisms* [in Russian], Nauka, Moscow (1988).
5. A. Tondl, *Nonlinear Oscillations of Mechanical Systems* [Russian translation], Mir, Moscow (1973).
6. I. B. Lebedenko, *Choice of the Parameters of Two-Chamber Pneumatic and Hydraulic Systems of Vibration Isolation with Interchamber Elements of Vibration Damping*, Author's Abstract of Candidate's Dissertation (in Engineering), Moscow (2002).